7.4., 14.5. - písemky (v F1)

↑ elektrostatika — el. obvody a magnetismus, střídavý proud

skripta 1991 Bakule a spol.: Příklady z el. a mag.

VEKTOROVÁ ANALÝZA

$|\vec{a}| = a$

$\vec{a}\cdot\vec{b} = a_1 b_1 + \dots = ab\cos\alpha = \vec{b}\cdot\vec{a}$

$\vec{a}(\vec{b}\cdot\vec{c}) = (\vec{a}\cdot\vec{b})\vec{c}$

$c = \vec{a}\times\vec{b} = \vec{\eta}\, ab\sin\alpha$

$c_x = a_2 b_3 - a_3 b_2$

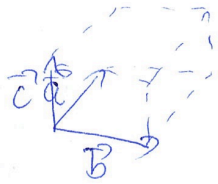
$c_y = a_3 b_1 - a_1 b_3$

$c_z = a_1 b_2 - a_2 b_1$

$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

$\vec{a}\times\vec{b} = -\vec{b}\times\vec{a}$

smíšený součin $\vec{c}\cdot(\vec{a}\times\vec{b})$

dejem 

$dl_i = k_i(q_1,q_2,q_3)\, dq_i$

↑ Lamèovy koef.

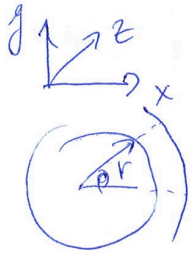
	i	k	l
dl_i	dx	dy	dz
dl_i	dr	dz	$r\,d\varphi$
dl_i	dr	$r\,d\upsilon$	$r\sin\upsilon\,d\varphi$

$dS = r\,dr\,d\varphi$

v sf. $dV = r^2\sin\upsilon\, dr\,d\upsilon\,d\varphi$

v kar. $dV = dx\,dy\,dz$





$\frac{x}{r} = \sin\upsilon$

$x = r\sin\upsilon$



$dl_r = dr$

$dl_\varphi = r\,d\varphi$

$\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$

gradient $\quad \operatorname{grad} f = \vec{F} = \nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right)$

divergence $\quad \operatorname{div}\vec{F} = g = \nabla\cdot\vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$

rotace (curl) $\quad \operatorname{rot}\vec{F} = \vec{G} = \nabla\times\vec{F} = \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}, \frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}, \frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)$

$\left(\frac{\partial}{\partial x}\right)\left(\frac{F_x}{F_y}\right)$
$\left(\frac{\partial}{\partial y}\right)\left(\frac{F_y}{F_z}\right)$
$\left(\frac{\partial}{\partial z}\right)$

Laplaceův op. $\quad \Delta = \nabla\cdot\nabla = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

$\vec{F} = (2y, -2x, 0)$

$\operatorname{rot}\operatorname{grad} f = 0 = \nabla\times\nabla f$

$\operatorname{div}\operatorname{rot}\vec{G} = 0$

$\operatorname{rot}\vec{F} = 0 \Rightarrow$ ex. $\vec{F} = \operatorname{grad} f$ — pole je potenciální

$\operatorname{div}\vec{G} = 0 \Rightarrow$ ex. $\vec{G} = \operatorname{rot}\vec{G}$ — solenoidní pole



Stokesova v. $\oint \vec{F} \cdot d\vec{l} = \int \mathrm{rot}\, \vec{F} \cdot d\vec{S}$

Gaussova v. $\oint \vec{F} \cdot d\vec{S} = \int \mathrm{div}\, \vec{F}\, dV$

Př. Spočtěte divergenci a rotaci vektorového pole. Je-li $\mathrm{rot}\,\vec{F}=0$, najděte sk. pole takové, aby $\vec{F} = \mathrm{grad}\, f$

- $\vec{F} = (2y,\ 2x+3z,\ 3y)$

$$\mathrm{div}\,\vec{F} = \nabla \cdot \vec{F} = \left(\frac{\partial 2y}{\partial x} + \frac{\partial (2x+3z)}{\partial y} + \frac{\partial 3y}{\partial z}\right)$$

$$= 0+0+0 = 0$$

$$\mathrm{rot}\,\vec{F} = \nabla \times \vec{F} = \left(\frac{\partial 3y}{\partial y} - \frac{\partial (2x+3z)}{\partial z},\ \frac{\partial 2y}{\partial z} - \frac{\partial 3y}{\partial x},\ \frac{\partial (2x+3z)}{\partial x} - \frac{\partial 2y}{\partial y}\right)$$

$$\begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} \begin{pmatrix} 2y \\ 2x+3z \\ 3y \end{pmatrix} = (0,\ 0,\ 0)$$

$$\vec{F} = \mathrm{grad}\, f = \nabla f = \left(\frac{\partial f}{\partial x},\ \frac{\partial f}{\partial y},\ \frac{\partial f}{\partial z}\right)$$

$3y = \frac{\partial f}{\partial z}$ $\quad 2y = \frac{\partial f}{\partial x}$ $\quad 2x+3z = \frac{\partial f}{\partial y}$

$\int 3y\, \partial z = \int \partial f$ $\quad \int 2y\, \partial x = \int \partial f$ $\quad (2x+3z)\partial y = \partial f$

$3yz = f_z$ $\quad 2yx = f_x$ $\quad \int (2x+3z)\partial y = \int \partial f$

3. 1. 2.

$(2x+3z)y = f_y$

$f = 2xy + 3yz$

$= 4xy + 6yz$

$= 2xy + 3yz$

Spočtěte gradient pole.

$\vec{r} = (x, y, z)$ $\quad r = \sqrt{x^2+y^2+z^2}$

- $\mathrm{grad}\, r = \left(\frac{\partial r}{\partial x},\ \frac{\partial r}{\partial y},\ \frac{\partial r}{\partial z}\right) = \left(\frac{1 \cdot 2x}{2\sqrt{x^2+y^2+z^2}},\ \frac{y}{\sqrt{x^2+y^2+z^2}},\ \frac{z}{\sqrt{--}}\right)$

$= \frac{\vec{r}}{r}$

- $\mathrm{grad}\, r^2 = \left(\frac{\partial r^2}{\partial x},\ \frac{\partial r^2}{\partial y},\ \frac{\partial r^2}{\partial z}\right) = (2x, 2y, 2z) = 2\vec{r}$
- $\mathrm{grad}\, \frac{1}{r} = \left(\frac{\partial \frac{1}{r}}{\partial x}, \ldots\right) = -\frac{1}{r^3} \cdot \vec{r}$

$$\frac{2 \cdot \frac{1}{2\sqrt{x^2+y^2+z^2}}}{2x} \quad / (x^2+y^2+z)^{-\frac{1}{2}}$$

$$= -\frac{1}{2}\left(x^2+y^2+z\right)^{-\frac{3}{2}} \cdot 2x = -\left(x^2+y^2+z\right)^{-\frac{3}{2}} \cdot x$$

- $\mathrm{grad}\, \frac{1}{r^3} = -\frac{3\vec{r}}{r^5}$

$$\frac{\partial \frac{1}{r^3}}{\partial x} = \frac{\partial (x^2+y^2+z^2)^{-\frac{3}{2}}}{\partial x} = -\frac{3}{2}(x^2+y^2+z^2)^{-\frac{5}{2}} \cdot 2x = -3x \cdot r^{-\frac{5}{2}}$$

gauss-gauss $\oint f d\vec{S} = \int \nabla f dV$

$\vec{r} = (x,y,z)$ $\quad r = (x^2+y^2+z^2)^{\frac{1}{2}}$ $\quad \vec{c} = (c_1, c_2, c_3)$

$\nabla(\vec{c}\cdot\vec{r}) = \nabla(c_1x + c_2y + c_3z) = (c_1, c_2, c_3) = \vec{c}$

$\nabla\left(\frac{\vec{c}\cdot\vec{r}}{r}\right)_x = \frac{1}{r}\cdot\nabla(\vec{c}\cdot\vec{r}) + \nabla\left(\frac{1}{r}\right)\cdot(\vec{c}\cdot\vec{r}) = \frac{c}{r} - \frac{\vec{r}}{r^3}(\vec{c}\cdot\vec{r}) = \frac{r^2\vec{c} - (\vec{c}\cdot\vec{r})\cdot\vec{r}}{r^3}$

Př. 4.

$\operatorname{div}\vec{r} = \left(\frac{\partial x}{\partial x}, \frac{\partial y}{\partial y}, \frac{\partial z}{\partial z}\right) = (1+1+1) = 3$

$\operatorname{rot}\vec{r} = \begin{pmatrix}\frac{\partial}{\partial x}\\ \frac{\partial}{\partial y}\\ \frac{\partial}{\partial z}\end{pmatrix} \times \begin{pmatrix}x\\ y\\ z\end{pmatrix} = \left(\frac{\partial z}{\partial y} - \frac{\partial y}{\partial z}, \dots, \dots\right) = \vec{0}$

$\operatorname{div}\frac{\vec{r}}{r} = \nabla\cdot\frac{\vec{r}}{r} = \frac{\partial \frac{x}{(x^2+y^2+z^2)^{\frac{1}{2}}}}{\partial x} + \frac{\partial \frac{y}{(x^2+y^2+z^2)^{\frac{1}{2}}}}{\partial y} + \frac{\partial \frac{z}{(x^2+y^2+z^2)^{\frac{1}{2}}}}{\partial z} = r^{-3}\left(r^2 - x^2 + r^2 - y^2 + r^2 - z^2\right)$

$= 3r^{-1} - r^3\left(x^2+y^2+z^2\right) = 3r^{-1} - r^{-3}\cdot r^2 = 3r^{-1} - r^{-1} = 2r^{-1}$

$\frac{\partial \frac{x}{(x^2+y^2+z^2)^{\frac{1}{2}}}}{\partial x} = \frac{1\cdot(x^2+y^2+z^2)^{\frac{1}{2}} - x\cdot\frac{1}{2}(x^2+y^2+z^2)^{-\frac{1}{2}}\cdot 2x}{x^2+y^2+z^2} = (x^2+y^2+z^2)^{-\frac{1}{2}} - x^2(x^2+y^2+z^2)^{-\frac{3}{2}}$

$\operatorname{rot}\frac{\vec{r}}{r} = \nabla\times\frac{\vec{r}}{r} = \frac{1}{r}(\nabla\times\vec{r}) + (\nabla\frac{1}{r})\times\vec{r} = \frac{1}{r}\begin{pmatrix}0\\0\\0\end{pmatrix} + \frac{\vec{r}}{r^3}\times\vec{r} = \vec{0}$

$\operatorname{div}\frac{\vec{r}}{r^3} = \nabla\cdot\frac{\vec{r}}{r^3} = \frac{1}{r^3}(\nabla\cdot\vec{r}) + (\nabla\frac{1}{r^3})\cdot\vec{r} = \frac{1}{r^3}\cdot 3 - \frac{3\vec{r}}{r^5}\cdot\vec{r} = \frac{3}{r^3} - \frac{3r^2}{r^5} = \frac{3}{r^3} - \frac{3}{r^3} = 0$

$\operatorname{rot}\frac{\vec{r}}{r^3} = \nabla\times\frac{\vec{r}}{r^3} = (\nabla\frac{1}{r^3})\times\vec{r} + (\nabla\times\vec{r})\cdot\frac{1}{r^3} = -\frac{3\vec{r}}{r^5}\times\vec{r} + 0\cdot\frac{1}{r^3} = -\vec{0}$

ELEKTROSTATIKA

Př. 1. 2 stejné částice náboje e $\quad m = ?$, aby $\vec{F}_G = -\vec{F}_{ee}$.

$\frac{1}{4\pi\varepsilon_0}\cdot\frac{e^2}{r^2} = G\frac{m^2}{r^2}$

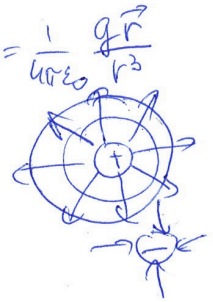
$m = \sqrt{\frac{e^2}{4\pi\varepsilon_0 G}} = 1{,}86\cdot10^{-9}\,kg$

$\frac{m}{m_{ee}} = 2\cdot10^{21}$

$\vec{F} = q\vec{E}$ $\quad \vec{E} = \frac{1}{4\pi\varepsilon_0}\sum\frac{q_i\vec{r}_i}{r^3}$ $\quad$ 1 náboj: $E = \frac{1}{4\pi\varepsilon_0}\frac{q\vec{r}}{r^3}$

$\vec{E}$ ↑ intenzita $\quad E = -\nabla\varphi$ ← potenciál $\quad \varphi = \frac{q}{4\pi\varepsilon_0}\cdot\frac{1}{r}$

$\varphi = -\nabla f$



Př. 11.9. $\frac{Q_2}{Q_1} = n$

$\varphi = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1}{\sqrt{x^2+y^2+z^2}} - \frac{nQ_1}{\sqrt{(x-d)^2+y^2+z^2}}\right) = 0$

$\frac{Q_1}{\sqrt{x^2+y^2+z^2}} = \frac{nQ_1}{\sqrt{(x-d)^2+y^2+z^2}}$

$$\sqrt{(x-d)^2+y^2+z^2} = n\sqrt{x^2+y^2+z^2}$$

$$(x-d)^2+y^2+z^2 = n^2(x^2+y^2+z^2)$$

$$x^2-2xd+d^2+y^2+z^2 = n^2x^2+n^2y^2+n^2z^2$$

$$x^2(n^2-1)+2xd+d^2+y^2(n^2-1)+z^2(n^2-1) = 0$$

$$x^2(n^2-1)+y^2(n^2-1)+z^2(n^2-1) = -2xd+d^2$$

$$x^2+y^2+z^2 = \frac{d^2-2xd}{n^2-1}$$

$$\left(x+\frac{2xd}{n^2-1}\right)^2 + \frac{2xd}{n^2-1} - \frac{4d^2}{(n^2-1)^2} + y^2+z^2 = \frac{d^2-2xd}{n^2-1}$$

$$\left(x+\frac{2d}{n^2-1}\right)^2 + y^2+z^2 = \frac{4d^2}{(n^2-1)^2} + \frac{d^2-2xd}{n^2-1} + \frac{2xd}{n^2-1}$$

$$= \frac{nd^2+d^2}{(n^2-1)^2} = \frac{d^2(d^2+n)}{(n^2-1)^2}$$

$$\varphi(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{|\vec{r}-\vec{r}'|} \qquad E(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{Q(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}$$

$$\sqrt{1+x} = 1+\frac{x}{2} \quad |x|\ll 1 \qquad \frac{1}{1-x} = 1+x \quad |x|\ll 1$$

$$\int_\Omega \nabla\cdot\vec{F}\,d\Omega = \int_{\partial\Omega}\vec{F}\cdot d\vec{S}$$

Gaussova věta

$\vec{p} = q\vec{l}\,\vec{e}_y$ dipólový moment

Vyšetřuj se $\varphi(\vec{r})$ a $E(\vec{r})$

$|\vec{r}| \gg l$.

Př.

dipól





$$\varphi(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{|\vec{r}-\vec{r}_+|}+\frac{-q}{|\vec{r}-\vec{r}_-|}\right) = \frac{q}{4\pi\varepsilon_0}\left(\frac{|\vec{r}-\vec{r}_-|-|\vec{r}-\vec{r}_+|}{|\vec{r}-\vec{r}_+|\cdot|\vec{r}-\vec{r}_-|}\right) \sim r^2$$

$$\simeq \frac{q}{4\pi\varepsilon_0}\left(\frac{\sqrt{x^2+(y+\frac{l}{2})^2}-\sqrt{x^2+(y-\frac{l}{2})^2}}{\sqrt{x^2+(y+\frac{l}{2})^2}\cdot\sqrt{x^2+(y-\frac{l}{2})^2}}\right) \sim r^2$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{\sqrt{1+\left(\frac{y+\frac{l}{2}}{x}\right)^2}-\sqrt{1+\left(\frac{y-\frac{l}{2}}{x}\right)^2}}{x^2}\right) \sim r^2$$

$$= \frac{q\sqrt{x^2+y^2}}{4\pi\varepsilon_0}\cdot\frac{1+\left(\frac{y+\frac{l}{2}}{x}\right)^2-1-\left(\frac{y-\frac{l}{2}}{x}\right)^2}{2r^2}$$

$$= \frac{qx^2}{4\pi\varepsilon_0}\cdot\frac{\frac{y^2+yl+\frac{l^2}{4}}{x^2}-\frac{y^2-yl+\frac{l^2}{4}}{x^2}}{2r^2}$$

$$= \frac{q\sqrt{x^2+y^2}}{4\pi\varepsilon_0}\cdot\frac{\sqrt{1+\frac{yl+\frac{l^2}{4}}{x^2+y^2}}-\sqrt{1+\frac{-yl+\frac{l^2}{4}}{x^2+y^2}}}{r^2}$$

$$= \frac{q}{4\pi\varepsilon_0}\cdot\frac{yl}{r^3} = \frac{qyl}{4\pi\varepsilon_0 r^2}\cdot\frac{r}{r} = \frac{q\vec{l}_j\cdot\vec{r}\cdot l}{4\pi\varepsilon_0 r^3} = \frac{\vec{p}\cdot\vec{r}}{4\pi\varepsilon_0 r^3}$$

$y = \vec{l}_j\cdot\vec{r}$

$$\frac{q\sqrt{x^2+y^2+yl+\frac{l^2}{4}}-\sqrt{x^2+y^2-yl+\frac{l^2}{4}}}{4\pi\varepsilon_0}\cdot\frac{r^2}{r^2}$$

$$\frac{q}{4\pi\varepsilon_0}\cdot\frac{1+\frac{yl+\frac{l^2}{4}}{r^2}\cdot\frac{1}{2}-1+\frac{yl-\frac{l^2}{4}}{2r^2}}{r^2}$$

$$\vec{E} = -\vec{\nabla}\varphi = -\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \cdot \frac{\vec{p}\cdot\vec{r}}{4\pi\varepsilon_0 r^3} = -\left(\frac{2\vec{p}\cdot\vec{r}}{4\pi\varepsilon_0 r^3}\right) \frac{\partial\left(\frac{\vec{p}\cdot\vec{r}}{4\pi\varepsilon_0 r^3}\right)}{\partial x}, \frac{\partial\left(\frac{\vec{p}\cdot\vec{r}}{4\pi\varepsilon_0 r^3}\right)}{\partial y}, \frac{\partial\left(\frac{\vec{p}\cdot\vec{r}}{4\pi\varepsilon_0 r^3}\right)}{\partial z}\Bigg) \quad (3)$$

$r = \sqrt{x^2+y^2+z^2}$ $\qquad \vec{p} = q l \vec{e}_y$

$$\varphi = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q l y}{r^3} \cdot \frac{1}{r^2} = \frac{q l y}{4\pi\varepsilon_0 \cdot (x^2+y^2+z^2)^{\frac{3}{2}}}$$

$$-\nabla\varphi_x = \left(+\frac{+ q l y \cdot 4\pi\varepsilon_0 \frac{3}{2}(x^2+y^2+z^2)^{\frac{1}{2}} \cdot 2x}{(4\pi\varepsilon_0 \cdot (x^2+y^2+z^2)^{\frac{3}{2}})^2}\right)$$

$$= \frac{q l y \, 4\pi\varepsilon_0 \cdot 3 \cdot r \cdot x}{4\pi\varepsilon_0 \cdot r^{2\frac{1}{2}}} = \frac{3 q l y x}{4\pi\varepsilon_0 r^5}$$

$$-\nabla\varphi_y = \frac{-3 q l y^2 - q l r^2}{4\pi\varepsilon_0 r^5}$$

$$\boxed{\vec{E} = \frac{3\vec{p}\cdot\vec{r}\,\vec{r} - \vec{p}r^2}{4\pi\varepsilon_0 r^5}}$$

Př. Skalární potenciál a intenzita dipólu v homogenním poli



$\vec{E} = (0,0,E_0)$

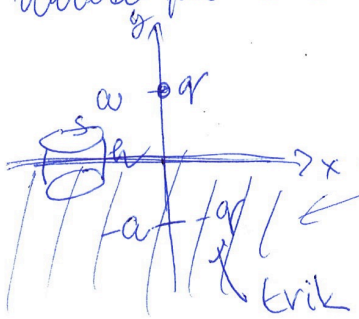
$\vec{p} = (0,0,p)$

Dokažte, že ekvipotenciální plochou s 0 pot. je koule a určete její poloměr.

$$\varphi_0 = -E_0 z \qquad \varphi = \frac{\vec{p}\cdot\vec{r}}{4\pi\varepsilon_0 r^3} - E_0 z = \frac{pz}{4\pi\varepsilon_0 r^3} - E_0 z = 0$$

$$a = \sqrt{x^2+z^2} = \left(\frac{p}{4\pi\varepsilon_0 E_0}\right)^{\frac{1}{3}}$$

Př. Určete pot. elstat. pole kodového náboje



$\varphi = ?$

$$\varphi(x,y) = \frac{q r}{4\pi\varepsilon_0} \cdot \left(\frac{1}{\sqrt{x^2+(y-a)^2}} - \frac{1}{\sqrt{x^2+(y+a)^2}}\right)$$

$$= \frac{q r}{4\pi\varepsilon_0}\left(\frac{(x,a)}{\sqrt{x^2+(y-a)^2}^3} - \frac{(x,a)}{\sqrt{x^2+(y+a)^2}^3}\right) \quad \text{Gauss}$$

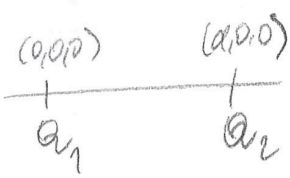
η plošná hustota náboje

$\vec{D}\cdot\vec{E} = \varepsilon \int$

$E_y = \sigma$

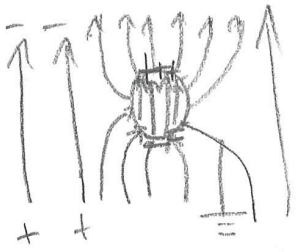
$\int \nabla\cdot\vec{E} = \int\int = \int E\cdot d\vec{S} \to E_y \cdot S = hS\rho = S\sigma$

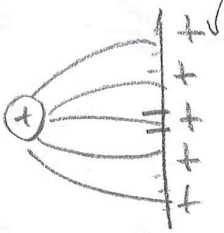
$\Rightarrow E_y = \sigma$



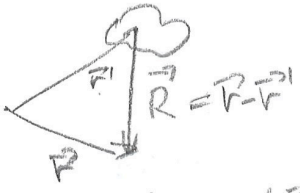
$\frac{Q_2}{Q_1} = n$ $\quad S = \left(-\frac{a}{n^2-1}, 0, 0\right)$

$a = \frac{an}{n^2-1}$





√ homogenně po úsečce ⊕ náboj z desky odlétá



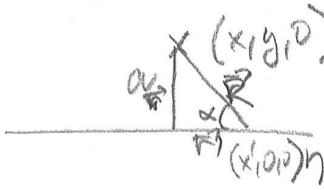
Př. $\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int_V \frac{\rho(\vec{r}')}{|\vec{r}-\vec{r}'|^3}\cdot(\vec{r}-\vec{r}')\,dV$

pro bodový náboj: $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^3}\cdot\vec{R}$

$\varphi(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(\vec{r}')}{|\vec{r}-\vec{r}'|}\,dV \qquad E = -\nabla\varphi$

σ ... dS plošná h.

η ... dl lineární hust.



$\frac{x'}{a} = \operatorname{cotg}\alpha \quad \frac{dx'}{a} = \frac{1}{-\sin^2\alpha}$

$\frac{a}{R} = \sin\alpha \Rightarrow R = \frac{a}{\sin\alpha}$

nekonečný vodič s lin. hust. náboje η

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int \frac{\eta}{|\vec{r}-\vec{r}'|^3}(\vec{r}-\vec{r}')\,d\vec{r}'$$

$$E_x = \frac{1}{4\pi\varepsilon_0}\int \frac{\eta}{|(x-x')|\,|\vec{r}-\vec{r}'|^3}(x-x')\,dx' = 0$$

$$E_y = \frac{1}{4\pi\varepsilon_0}\int \frac{\eta}{|\vec{r}-\vec{r}'|^3}(y)\,dx' = \frac{1}{4\pi\varepsilon_0}\int \frac{\eta}{|\vec{R}|^3}\cdot y\,dx'$$

$$= \frac{\eta}{4\pi\varepsilon_0}\int \frac{\sin^2\alpha}{a^2}\,a\,\frac{a}{-\sin^2\alpha}\,d\alpha = \frac{-\eta}{4\pi\varepsilon_0}\int \frac{\sin\alpha}{a}\,d\alpha$$

$$= \frac{-\eta}{4\pi\varepsilon_0 a}\int \sin\alpha\,d\alpha = \frac{+\eta}{4\pi\varepsilon_0}\Big[+\cos\alpha\Big]_0^\pi =$$

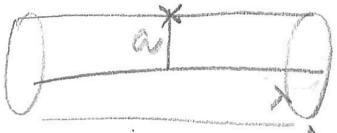
$$= \frac{+\eta}{4\pi\varepsilon_0}(+1+1) = \frac{+\eta}{2\pi\varepsilon_0 a}$$

pomocí Gaussovy věty: $\oint \vec{E}\cdot d\vec{S} = \frac{Q}{\varepsilon_0} \Leftrightarrow \oint E\,dS = \frac{Q}{\varepsilon_0}$ pro $\vec{E}\parallel d\vec{S}$

a pro E = konst $\Leftrightarrow \oint E\,dS = E\oint dS = ES = \frac{Q}{\varepsilon_0}$

$$E = \frac{Q}{\varepsilon_0 S} = \frac{Q}{\varepsilon_0 2\pi r l} = \frac{\lambda\cdot l}{\varepsilon_0 2\pi r l} = \frac{\lambda}{2\varepsilon_0\pi a}$$

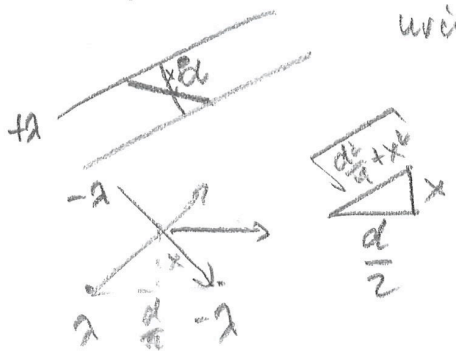
$S = 2\pi r l$



↖ ekvipot. plocha a $\vec{E}\parallel\vec{n}$

Př. (1.22) $\lambda = \text{konst}$

určete intenzitu v bodě ve výšce x



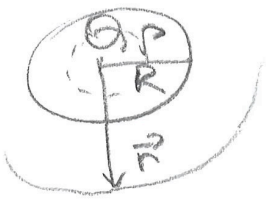
$$E = E_1 + E_2 = \frac{\lambda}{2\varepsilon_0 \pi \cdot \sqrt{\frac{d^2}{4}+x^2}} \cdot \frac{\begin{pmatrix} d/2 \\ x \end{pmatrix}}{\sqrt{\frac{d^2}{4}+x^2}} + \frac{+\lambda}{2\pi\varepsilon_0 \cdot \sqrt{\frac{d^2}{4}+x^2}} \cdot \begin{pmatrix} d/2 \\ -x \end{pmatrix} \cdot \frac{1}{\sqrt{\frac{d^2}{4}+x^2}}$$

$$= \frac{\lambda}{2\pi\varepsilon_0 \left(\frac{d^2}{4}+x^2\right)} \begin{pmatrix} d/2 \\ x \end{pmatrix} + \frac{+\lambda}{2\pi\varepsilon_0 \left(\frac{d^2}{4}+x^2\right)} \cdot \begin{pmatrix} d/2 \\ -x \end{pmatrix}$$

$$= \frac{\lambda}{2\pi\varepsilon_0 \left(\frac{d^2}{4}+x^2\right)} \left(\begin{pmatrix} d/2 \\ x \end{pmatrix} + \begin{pmatrix} d/2 \\ -x \end{pmatrix} \right) = \frac{\lambda}{2\pi\varepsilon_0 \left(\frac{d^2}{4}+x^2\right)} \begin{pmatrix} d \\ 0 \end{pmatrix}$$

Př. (1.19) Rovnoměrně nabitá koule

↓ dielektrická



$\oint \vec{E} \cdot d\vec{S} = \frac{Q}{\varepsilon_0}$

opět je $\vec{n} \parallel \vec{E}$ ↗

$Q = \frac{4}{3}\rho\pi r^3$

$r > R$ - vně:

$ES = \frac{Q}{\varepsilon_0}$

$E = \frac{Q}{S\varepsilon_0} = \frac{Q}{4\pi r^2 \varepsilon_0}$

$= \frac{-Q}{4\pi\varepsilon_0 r^2} \cdot \frac{\vec{r}}{r}$

$r < R$ - uvnitř:

$q = \frac{Q \frac{4}{3}\rho\pi r^3}{\frac{4}{3}\rho\pi R^3} = Q\frac{r^3}{R^3}$

$E = \frac{Q^{\nearrow q}}{S\varepsilon_0} = \frac{Q\frac{r^3}{R^3}}{4\pi r^2 \varepsilon_0} = \frac{Qr}{4\pi R^3 \varepsilon_0}$

potenciál uvnitř: $r > R$

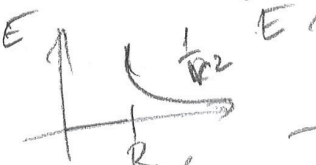
$\vec{E} = -\nabla\varphi$

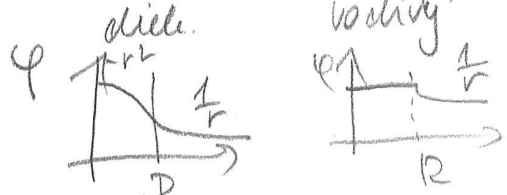
$\vec{E}(r) = -\frac{\partial\varphi}{\partial r}$

$\int \frac{+Q}{4\pi\varepsilon_0 r^2} = \int +\frac{\partial\varphi}{\partial r}$

$\frac{+Q}{4\pi\varepsilon_0} \int \frac{1}{r^2} dr = \int d\varphi$

$\varphi = \frac{C}{Q} \cdot r^{-1} = \frac{Q}{4\pi\varepsilon_0 r} = \frac{-Q}{4\pi\varepsilon_0 r} + C_1 \to 0$ pro $r \to \infty$ 0 pot.





potenciál uvnitř $r < R$:

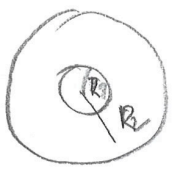
$\vec{E}(r) = -\frac{\partial\varphi}{\partial r}$

$\int \frac{Qr}{4\pi R^3 \varepsilon} = \int -\frac{\partial\varphi}{\partial r}$

$\varphi = -\frac{Q}{4\pi R^3 \varepsilon} \int r\,dr = -\frac{Q}{4\pi R^3 \varepsilon} \cdot \frac{r^2}{2} + C_2$

↓ pro $r = R$ $= -\frac{Q}{4\pi\varepsilon_0 r}$

Protože potenciál je vždy spojitý!



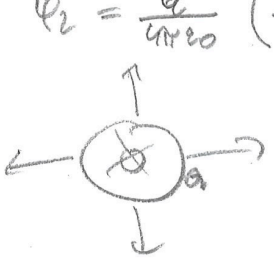
$$\varphi_1 = A_{11}Q_1 + A_{12}Q_2$$
$$\varphi_2 = A_{21}Q_1 + A_{22}Q_2$$
$$Q_1 = C_{11}\varphi_1 + C_{12}\varphi_2$$
$$Q_2 = C_{21}\varphi_1 + C_{22}\varphi_2$$

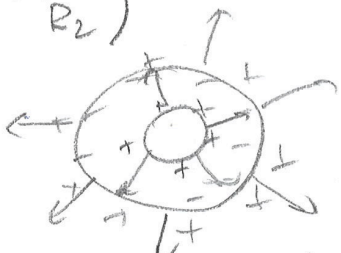
$\varphi = AQ$ … A je nějaká konst. ↘ influenční koef.

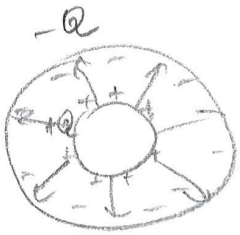
$C\varphi = CAQ$ (I) $\Rightarrow C\varphi = Q$

$Q = Q_1 - Q_2$ ↖ kapacitní koef.

$$\varphi_1 = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$
$$\varphi_2 = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R_2} - \frac{1}{R_2}\right)$$







$$\left(\begin{array}{cc|cc} \frac{1}{R_1} & \frac{1}{R_2} & 1 & 0 \\ \frac{1}{R_2} & \frac{1}{R_2} & 0 & 1 \end{array}\right) \sim \left(\begin{array}{cc|cc} 1 & 0 & \frac{R_1R_2}{R_2-R_1} & -\frac{R_1R_2}{R_2-R_1} \\ 0 & 1 & -\frac{R_1R_2}{R_2-R_1} & \frac{R_2}{R_2-R_1} \end{array}\right)$$

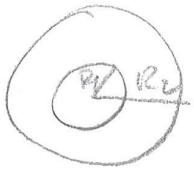
influenční k. … kapacitní k.

$C_{12} = C_{21}$ (symetrie)

$C_{11} = -C_{12}$ podle kondenzátoru

KAPACITA $C = \frac{Q}{U}$ $\quad U = \varphi_2 - \varphi_1$

Př. 1.2.6. kondenzátor … kapacita pomocí kap. a infl. koeficientů



$$U = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$C = \frac{Q}{U} = \frac{Q}{\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)} = 4\pi\varepsilon_0 \frac{R_2 R_1}{R_2 - R_1}$$

$$U = \varphi_A - \varphi_B = Q\left(\frac{C_{11} - C_{12} - C_{11} + C_{21}}{C_{11}C_{22} - C_{12}C_{21}}\right)$$

Př. 1.2.7.

$$\oint \vec{E}\cdot d\vec{S} = \frac{Q}{\varepsilon_0} \qquad S = 2\pi r l$$

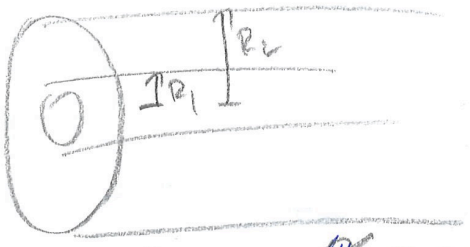


$$SE = \frac{Q}{\varepsilon_0}$$
$$E = \frac{Q}{2\pi r l \varepsilon_0} = \frac{\sigma \cdot 2\pi R l}{2\pi\varepsilon_0 r l} = \frac{\sigma R}{r\varepsilon_0}$$

$E = -\nabla\varphi$ … φ … $\int E = \int -\frac{\partial\varphi}{\partial r}$

$$\int \frac{\sigma R}{r\varepsilon_0}\,dr = -\int d\varphi$$
$$\varphi = -\frac{\sigma R}{\varepsilon_0}\ln|r| + c = \frac{\sigma R}{\varepsilon_0}\ln\frac{1}{r} + c$$

nemůže být 0 pot. v ∞ → bude na elektrodě $\Rightarrow c = \frac{\sigma R}{\varepsilon_0}\ln\frac{1}{R}$ ↓ $\varphi = \frac{\sigma R}{\varepsilon_0}\ln\left(\frac{R}{r}\right)$



$$\varphi_1 = \frac{\sigma_1 R_1}{\varepsilon_0}\ln\frac{R_1}{R_1} + \frac{\sigma_2 R_2}{\varepsilon_0}\ln\frac{R_1}{R_2}$$

$$\varphi_2 = \frac{\sigma_1 R_1}{\varepsilon_0}\ln\frac{R_1}{R_2} + \frac{\sigma_2 R_2}{\varepsilon_0}\ln\frac{R_1}{R_2} \quad \leftarrow \text{integr. konst.}$$

$$U = \varphi_2 - \varphi_1 = \frac{\sigma_1 R_1}{\varepsilon_0}\ln\frac{R_2}{R_1}$$

$$C = \frac{Q}{U} = \frac{\sigma_1 R_1 2\pi l}{-\frac{\sigma_1 R_1}{\varepsilon_0}\ln\frac{R_1}{R_2}} = \frac{2\pi\varepsilon_0 l}{-\ln\left(\frac{R_1}{R_2}\right)}$$

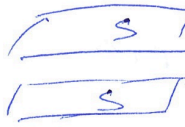
$$ES = \frac{Q}{\varepsilon_0} \qquad E\,2\pi r l = \frac{Q}{\varepsilon_0} \qquad E = \frac{Q}{2\pi\varepsilon_0 l}\cdot\frac{1}{r} \Rightarrow \varphi = -K\int\frac{1}{r} = K\log\frac{1}{r}+C$$

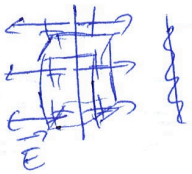
$$= K\log\frac{r_0}{r}$$

$$\varphi_1 - \varphi_2 = \frac{Q}{2\pi\varepsilon_0 l}\left(\ln\frac{R_1}{R_1} - \ln\frac{R_1}{R_2}\right) = \frac{Q}{2\pi\varepsilon_0 l}\log\frac{R_2}{R_1}$$

$$-\int_A^B \vec{E}\cdot d\vec{l} = U = -\int_{R_2}^{R_1}\frac{Q}{2\pi\varepsilon_0 l}\cdot\frac{1}{r}\,dr = -\frac{Q}{2\pi\varepsilon_0 l}\left[\log R_1 - \log R_2\right] = \frac{Q}{2\pi\varepsilon_0 l}\log\frac{R_2}{R_1}.$$

Př. (1.2.8) Kapacita deskového kond.





$$\oint \vec{E}\cdot d\vec{S} = \frac{Q}{\varepsilon_0}$$

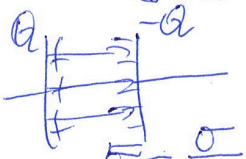
$$2E\cdot S = \frac{Q}{\varepsilon_0} \qquad C = \frac{Q}{U}$$

$$= \frac{\sigma S}{\varepsilon_0} \Rightarrow E = \frac{\sigma}{2\varepsilon_0}$$

$$\varphi = -\int\frac{\sigma}{2\varepsilon_0}\,dx \overset{C}{=} -\frac{\sigma}{2\varepsilon_0}x$$

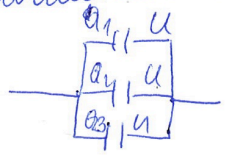
$$U = -\int_d^0 \frac{\sigma}{\varepsilon_0}\,dx = -\frac{\sigma}{\varepsilon_0}(0-d)$$

$$= \frac{d\sigma}{\varepsilon_0}$$



$$E = \frac{\sigma}{\varepsilon_0} \quad \left(2\times\frac{\sigma}{2\varepsilon_0}\right) \qquad C = \frac{Q}{U} = \frac{\sigma S}{\frac{d\sigma}{\varepsilon_0}} = \frac{\varepsilon_0 S}{d}$$

Př. Kapacita při sériovém a paralelním zapojení



$$Q = Q_1 + Q_2 + Q_3 \qquad C_1 = \frac{Q_1}{U} \quad C_2 = \frac{Q_2}{U} \quad C_3 = \frac{Q_3}{U}$$

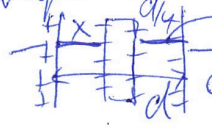
$$C = \frac{Q}{U} = \frac{Q_1+Q_2+Q_3}{U} = \frac{Q_1}{U} + \frac{Q_2}{U} + \frac{Q_3}{U} = C_1 + C_2 + C_3$$



$$U = U_1 + U_2 + U_3 \qquad Q = Q_1 = Q_2 = Q_3$$

$$\frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} \Rightarrow \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Př. (1.2.11) Deskový kondenzátor s kapacitou C. Jaká bude kapacita po vložení plechu?

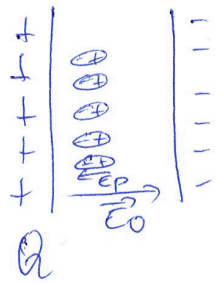


dva sériově zapojené kondenzátory

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{x}{S\varepsilon_0} + \frac{d-x-\frac{d}{4}}{S\varepsilon_0} =$$

$$= \frac{d-x-\frac{d}{4}+x}{S\varepsilon_0} = \frac{\frac{3}{4}d}{S\varepsilon_0} \Rightarrow C = \frac{4}{3}\frac{S\varepsilon_0}{d}$$

Př.



$\vec{E} = \vec{E}_0 \rightarrow \vec{E} = \vec{E}_0 + \vec{E}_p$

$\perp x$: $E = E_0 - E_p$

$E = \frac{\sigma}{\varepsilon_0} - \frac{\sigma_p}{\varepsilon_0}$

$\varepsilon_0 E = \sigma - \sigma_p$

$\varepsilon_0 E = \sigma - P$

$\varepsilon_0 E = \sigma - \chi \varepsilon_0 E$

$\varepsilon E (1+\chi) = \sigma$ ($1+\chi = \varepsilon_r$)

$\sigma_p = \vec{P} \cdot \vec{n}$

$P = \chi \varepsilon_0 E$

$\rightarrow E \varepsilon_0 \varepsilon_r = \sigma$ ($\varepsilon_0 \varepsilon_r = \varepsilon$)

$E \varepsilon = \sigma$

$D = \sigma$ / S (vektor el. indukce)

$\oint \vec{D} \cdot d\vec{S} = Q$

Př. (1.3.1) Deskový kondenzátor o kap. C_0, plošě S, náboj Q. Přidali jsme dielektrikum a $C_0 \rightarrow C$. Spočtěte E, σ_p

Kond. není připojen ke zdroji $\Rightarrow Q$ = konst $C = \frac{\varepsilon_0 S}{d}$

$\vec{E}$ před vložením diel.

$Ed = U = \frac{Q}{C} \Rightarrow E_0 = \frac{Q}{Cd} = \frac{Q}{\frac{\varepsilon_0 S}{d} \cdot d} = \frac{Q}{\varepsilon_0 S} = \frac{\sigma S}{\varepsilon_0 S} = \frac{\sigma}{\varepsilon_0}$

po vložení:

$E = \frac{\sigma}{\varepsilon_0 \varepsilon_r} \Rightarrow \frac{E}{E_0} = \frac{\frac{\sigma}{\varepsilon_0 \varepsilon_r}}{\frac{\sigma}{\varepsilon_0}} = \frac{1}{\varepsilon_r}$

$\varepsilon_r = \chi + 1$

$\chi = \varepsilon_r - 1$

σ_p: $-\sigma_p = \vec{P} \vec{n} = P = \chi \varepsilon_0 E$

$= (\varepsilon_r - 1) \varepsilon_0 E = (\varepsilon_r - 1) \varepsilon_0 \frac{E_0}{\varepsilon_r}$

$= (\varepsilon_r - 1) \varepsilon_0 \cdot \frac{\sigma}{\varepsilon_0 \varepsilon_r} = \frac{\sigma(\varepsilon_r - 1)}{\varepsilon_r}$

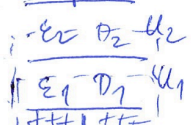
U: $U = E \cdot d = \frac{\sigma}{\varepsilon_0 \varepsilon_r} d = \frac{E_0}{\varepsilon_r} d$

při U = konst:

$\sigma = \sigma_0 \varepsilon_r$

σ_p: $\sigma_p = P = \chi \varepsilon_0 E = (\varepsilon_r - 1) \varepsilon_0 \cdot E_0$

Př. (1.3.4)



$U = U_1 + U_2$

$D = D_1 = D_2$

$\varepsilon E = \varepsilon_1 E_1 = \varepsilon_2 E_2$

$D = \frac{Q}{S}$ $D = \varepsilon E$

$DS = Q$

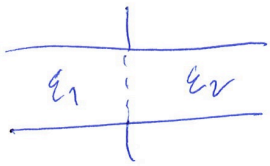
$D = E \varepsilon$

$\varepsilon_2 > \varepsilon_1$: $E_2 < E_1$

$U = E_1 d + E_2 d = \frac{D}{\varepsilon_1} d + \frac{D}{\varepsilon_2} d = D\left(\frac{d}{\varepsilon_1} + \frac{d}{\varepsilon_2}\right) = \frac{Q}{S}\left(\frac{d}{\varepsilon_1} + \frac{d}{\varepsilon_2}\right) = Q\left(\frac{d}{S\varepsilon_1} + \frac{d}{S\varepsilon_2}\right)$

$\frac{1}{C} = \frac{Q}{U} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d}{S\varepsilon_1} + \frac{d}{S\varepsilon_2}$

Př.



$U_1 = U_2 = U = Ed$

$CU = Q = Q_1 + Q_2 = D_1 \frac{S}{2} + D_2 \frac{S}{2} = \frac{\varepsilon_1 E S}{2} + \frac{\varepsilon_2 E S}{2}$

$= \frac{Ed\varepsilon_1 S}{2d} + \frac{Ed\varepsilon_2 S}{2d} \Rightarrow C = C_1 + C_2$

(C_1: U, C_2: U)

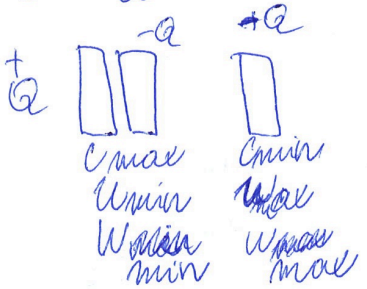
Př. (Nyní)

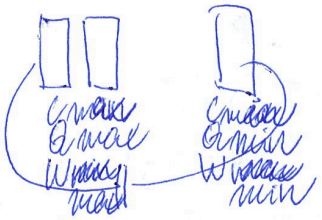
Energie kondenzátoru

$$W = \int_0^Q U\,dQ = \frac{1}{C}\int_0^Q Q\,dQ = \frac{1}{2C}Q^2 = \frac{1}{2}QU = \frac{1}{2}CU^2$$

$C = \frac{\varepsilon S}{d}$ $\quad C = \frac{Q}{U}$ $\quad D = \varepsilon E$ $\quad Q = DS$

$F = -\operatorname{grad} W$





⇒ nabíjení musí vykonat vyšší práci na donesení náboje

Př. (14.1) otočný kondenzátor

min. kapacita C_0. Jakou W vykonáme při změně $C_{max} \to C_0$, jestliže na elektrodách je U = konst?



$W_0 = \frac{1}{2}C_0U^2 \qquad W_{max} = \frac{1}{2}C_{max}U^2$

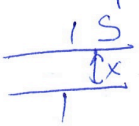
$\Rightarrow \delta W = \frac{1}{2}(C_{max} - C_0)U^2$

Jakou práci vykonáme, jestliže bez nabití na napětí U a při otáčení zdroj odpojíme?

$W_0 = \frac{1}{2}\frac{Q^2}{C_0} \qquad W_{max} = \frac{1}{2}\frac{Q^2}{C_{max}}$

$\delta W = \frac{1}{2}\left(\frac{Q^2}{C_0} - \frac{Q^2}{C_{max}}\right) = \frac{Q^2}{2}\left(\frac{1}{C_0} - \frac{1}{C_m}\right) = \frac{Q^2}{2}\frac{C_m - C_0}{C_0 C_m}$

Př. Jaká síla působí na elektrony desti. kond. nabitého na nap. U, plocha je S, vzd. x.



$C = \frac{\varepsilon S}{x} \qquad F = -\operatorname{grad} W$

$W = \frac{1}{2}CU^2 = \frac{1}{2}\frac{\varepsilon S}{x}U^2$

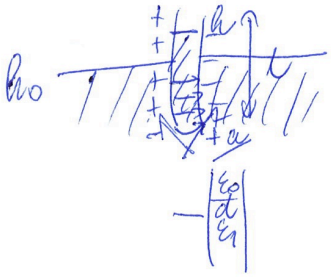
$F_x = -\frac{1}{2}\frac{\varepsilon S}{x}U^2 \cdot \frac{\partial}{\partial x} = +\frac{1}{2}\varepsilon S U^2 \cdot \left(+\frac{1}{x^2}\right) = \frac{1}{2}\frac{\varepsilon S U^2}{x^2}$

$F_y = 0$

$F_z = 0$

$\vec{F} = \left(\frac{1}{2}\frac{\varepsilon S U^2}{x^2}, 0, 0\right)$

Př. (1.4.6) kond. ponořený do kapaliny
kap. vystoupá! proč?



$mgh = \frac{a}{2} cu^2$

$h = \frac{1}{2} \frac{cu^2}{mg} \rightarrow \frac{1}{2} \frac{cu^2}{\rho V g} = \frac{cu^2}{\rho S g}$

$W = \frac{1}{2}(C_1 + C_2) u^2 = \frac{1}{2}\left(\frac{\varepsilon_1 a \cdot}{d} + \frac{\varepsilon_0 (\cdot)}{d} a\right) u^2$

$C = \frac{\varepsilon S}{d}$ $= \frac{a}{2d}\left(\varepsilon_1 h - \varepsilon_0 (l-h)\right) u^2 = \frac{h \cdot a}{2d}\left(l(\varepsilon_1 - \varepsilon_0) + \varepsilon_0 h\right) u^2$

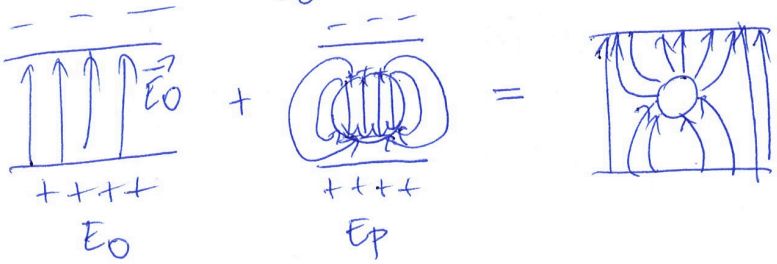
~~$mgh = \frac{a}{2d}(\ldots)$~~ derivujeme → grad → F

$mgh = F$

Př. bod. náboj: $W = QU$ (vnější pole)
dipól: $W = -\vec{p} \cdot \vec{E}_0$
dielektr. tuhé: $W = -\int \vec{P}_0 \vec{E}_0 \, dV$
diel. měkké: $W = -\int \vec{P}_m \vec{E}_0 dV + \frac{1}{2}\int \vec{P}_m \cdot \vec{E}_0 dV = -\frac{1}{2}\int \vec{P}_m \cdot \vec{E}_0 \, dV$

$\vec{E} = \vec{E}_0 + \vec{E}_p$ (výsl. ↑ pole epol. diel.)

Př. do homog. pole $\vec{E}_0$ byla vložena koule o pol. R. Spočtěte výsl. energii (jí z měkkého diel.) koule.



$W = -\frac{1}{2}\int \vec{P}_m \cdot \vec{E}_0 \, dV$

$\vec{P} = \varepsilon_0 \chi \vec{E}$

$\vec{E} = \vec{E}_0 + \vec{E}_p$

$\sigma_p = \vec{P} \cdot \vec{n} = P \cos\vartheta$

$E = \frac{1}{4\pi\varepsilon_0}\int \frac{\sigma}{R^2} dS$

$E_p = \frac{-P}{3\varepsilon_0}$

$E = E_0 + \frac{P}{3\varepsilon_0} = E_0 - \frac{\varepsilon_0 \chi E}{3\varepsilon_0}$

$E\left(1 - \frac{\varepsilon_0 \chi}{3\varepsilon_0}\right) = E_0 \Rightarrow E = \frac{E_0}{1 - \frac{\varepsilon_0 \chi}{3\varepsilon_0}} = \frac{E_0}{\frac{3-\chi}{3}} = \frac{3E_0}{3-\chi} =$

$= \frac{3E_0}{3 - (\varepsilon_r - 1)}$

$P = \frac{(\varepsilon - \varepsilon_0) 3 \varepsilon_0 E_0}{2\varepsilon_0 + \varepsilon}$

$W = -\frac{1}{2} \cdot \frac{4}{3}\pi R^3 \cdot \frac{(\varepsilon - \varepsilon_0) 3 \varepsilon_0 E_0}{2\varepsilon_0 + \varepsilon} \cdot E_0 = \frac{(\varepsilon_0 - \varepsilon)\varepsilon_0 E_0^2}{2\varepsilon_0 + \varepsilon}$

ELEKTRICKÝ PROUD, OBVODY.

Př. železný drát — jaká je V-A charakteristika variátoru? R závisí lineárně na teplotě.

$$R_t = R_0 (1 + \alpha t + \beta t^2 + \dots)$$

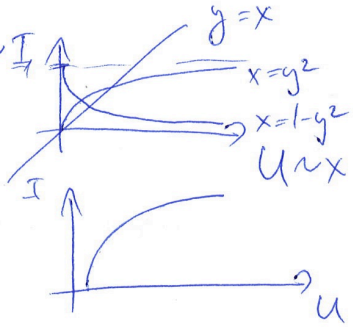
$$R = \frac{U}{I} = R_0 (1 + \alpha \Delta t) = R_0 (1 + \alpha g U I)$$

$$U (1 - R_0 I^2 \alpha g) = R_0 I$$

$$U = \frac{R_0 I}{1 + R_0 I^2 \alpha g}$$

$\sqrt{R_0 \alpha g}\, I = y$

$$U = \frac{R_0 I}{1 + y^2} \cdot \frac{\sqrt{\frac{\alpha g}{R_0}}}{\sqrt{\frac{\alpha g}{R_0}}} = \sqrt{\frac{\alpha g}{R_0}}\, U = x = \frac{y}{1 + y^2}$$



Př. deska — stanovte odpor mezikruží!



$$R = \frac{U}{I}$$

$E = \frac{\sigma}{\varepsilon_0} \cdot \frac{r_1}{r}$ – už víme

$$U = \frac{\sigma r_2}{\varepsilon_0} \ln \frac{r_2}{r_1} - \frac{\sigma r_1}{\varepsilon_0} \ln \frac{r_1}{r_1} = \frac{\sigma r_2}{\varepsilon_0} \ln \frac{r_2}{r_1}$$

$$I = \int \vec{i} \cdot d\vec{S} = \gamma \int \vec{E} d\vec{S} = \gamma \frac{\sigma}{\varepsilon_0} \cdot \frac{r_1}{x} 2\pi x t = \dots \quad \sigma_1 r_1 = Q = \sigma_2 r_2$$

$\vec{i} = \gamma \vec{E}$

$i\, dS = \frac{1}{\frac{dx}{\gamma dS}} E\, dx$ ($i = I$, $\frac{dx}{\gamma dS} = R$, $E\,dx = U$)

$$R = \frac{U}{I} = \frac{\frac{\sigma r_1}{\varepsilon_0} \ln \frac{r_2}{r_1}}{\gamma \frac{\sigma_1 r_1}{\varepsilon_0} 2\pi t} = \frac{\ln \frac{r_2}{r_1}}{\gamma 2\pi t}$$

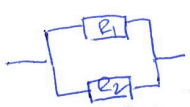
jinak: $dR = \frac{dx}{\gamma S(x)} \Rightarrow R = \frac{1}{\gamma} \int_{r_1}^{r_2} \frac{dx}{2\pi x t} = \frac{1}{\gamma 2\pi t} \underbrace{\int_{r_1}^{r_2} \frac{dx}{x}}_{\ln \frac{r_2}{r_1}}$

Př. 

I
U

sériové odpory $U = U_1 + U_2$ $I = I_1 = I_2$

$$R = \frac{U}{I} = \frac{U_1 + U_2}{I} = R_1 + R_2$$



$U = U_1 = U_2$

$I = I_1 + I_2$

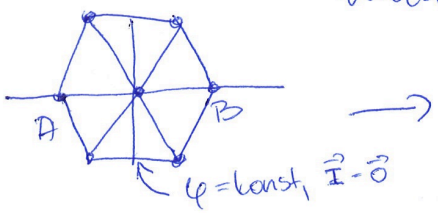
paralelní odpory

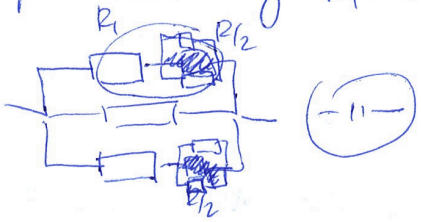
$$R = \frac{U}{I} = \frac{U}{I_1 + I_2}$$

$$R^{-1} = \frac{I_1 + I_2}{U} = \frac{I_1}{U} + \frac{I_2}{U} = R_1^{-1} + R_2^{-1}$$

Př. 5.1.4.

Určete odpor mezi body A, B.





$R_1 = \frac{4R}{3} = \frac{3}{2} + R$

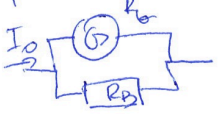
$\frac{1}{\frac{R_1}{2}} + \frac{1}{R} = \frac{3}{R}$

$\frac{2}{R_V} = \frac{6}{4R} + \frac{1}{R} = \frac{5}{2}R$

$\frac{1}{R} R_V = \frac{2}{5} R$ $\quad R_V = \frac{4}{5} R$

Př. 5.1.11. Ke Galvanometru s vnitřním odporem R_G je připojen bočník s odp. R_B, který 10x sníží citlivost Galvanometru. Jaký odpor obvod musíme připojit, aby to fungovalo?

$R = ?$ aby citlivost snížil 10x?



$I_G = \frac{1}{10} I_0$ $\quad U = R_G I_0 = R_1 I_1$

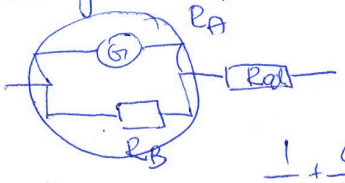
$I_0 = I_B + I_G \Rightarrow I_B = \frac{9}{10} I_0$

$R_B = \frac{U_B}{I_B} = \frac{U}{\frac{9}{10} I_0} \rightarrow \frac{10}{9} R_G$ (← pravda)

$I_B = 9 I_G$ $\quad R = \frac{U}{I}$

$\frac{U_B}{R_B} = 9 \frac{U}{R_G}$ $\rightarrow R_B = \frac{1}{9} R_G$

Připojíme ještě odpor tak, aby R bylo stejné jako R_G.



$\frac{1}{R_G} + \frac{1}{R_B} = \frac{1}{R_A}$

$\frac{1}{R_G} + \frac{9}{R_G} = \frac{1}{R_A} \Leftrightarrow \frac{1}{R_G} + \frac{1}{\frac{1}{9} R_G} = \frac{1}{R_A}$ $\quad \frac{1}{10}$

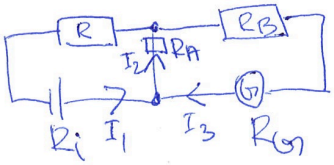
$\frac{10}{R_G} = \frac{1}{R_A} \rightarrow R_A = \frac{9}{10} R_G$

$R_{od} + \frac{9}{10} R_G = R_G$

$R_{od} = \frac{9}{10} R_G$.

Kirchoffovy zákony

Př. 5.1.16



g uzlů, poč. nezávislých je g − 1

$\sum I_i = 0$ $\quad I_1 + I_3 = I_2$

n větví, e smyček, poč. nezávislých je n − (g − 1)

$\sum R_i I_i = \sum \varepsilon_i$ ← součet napětí

$I_1 (R + R_i) + I_2 R_A = \mathcal{E}$

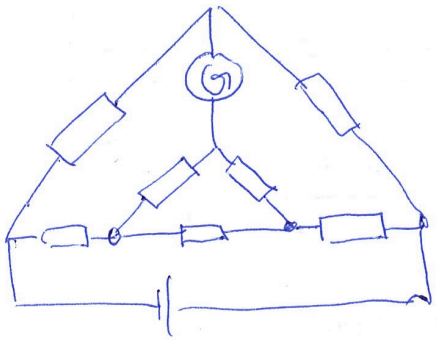
$I_3 (R_B + R_G) + I_2 R_A = 0$

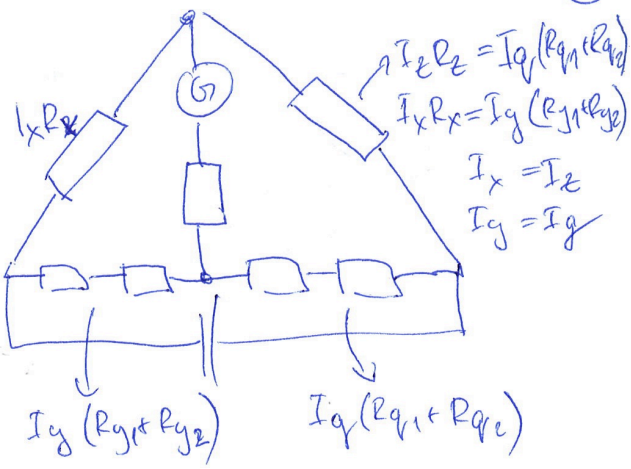


$I_{01} (R + R_i + R_A) + I_{02} R_A = \mathcal{E}$

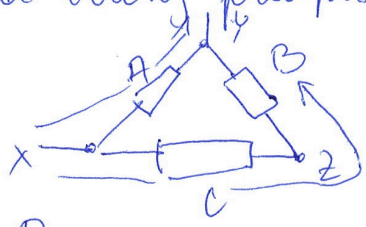
$I_{02} (R_A + R_B + R_G) + I_{01} R_A = 0$

Př. 5.1.33 Určete podmínky rovnováhy na mostku $I_G = 0$ (5 b)





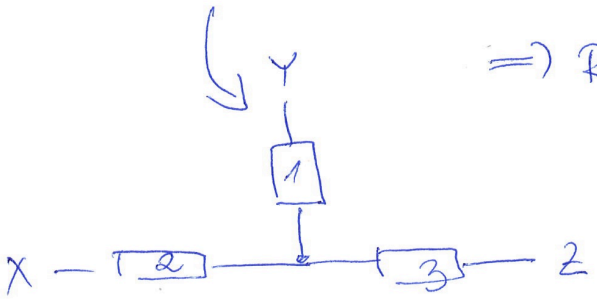
platí obvyklý předpis:



$$R_{XY} = R_1 + R_2$$

$$\frac{1}{R_{XY}} = \frac{1}{R_A} + \frac{1}{R_B + R_C} = \frac{R_A + R_B + R_C}{R_A (R_B + R_C)}$$

$$\Rightarrow R_{XY} = \frac{R_A (R_B + R_C)}{R_A + R_B + R_C}$$



$$R_{YZ} = R_1 + R_3$$

$$R_{XZ} = R_2 + R_3$$

$$R_1 = \frac{R_{XY} + R_{YZ} - R_{XZ}}{2}$$

Př. $\vec{E} = \frac{1}{4\pi\varepsilon_0} \int \frac{\varepsilon(\vec{r}')\,(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3}\, dV$

$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{\vec{i}(\vec{r}') \times (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3}\, dV \overset{i=\text{konst}}{=} \frac{\mu_0}{4\pi} \int \frac{\vec{i} \times \vec{R}}{R^3}\, dV \qquad I = \int \vec{i}\, d\vec{S}$$

$$= \frac{\mu_0 I}{4\pi} \int \frac{d\vec{l} \times \vec{R}}{R^3}\, dV$$

$$\vec{E} = -\nabla \varphi$$

$$\boxed{\vec{B} = \nabla \times \vec{A}}$$

Př. 3.1.1.



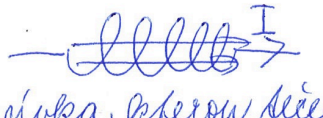
B ve středu kruhu

$$B = \frac{\mu_0 I_1}{4\pi} \int_{l_1} \frac{R \sin\frac{\pi}{2}}{R^3} dl + \frac{\mu_0 I_2}{4\pi} \int_{l_2} \frac{R \sin\frac{-\pi}{2}}{R^3} dl$$

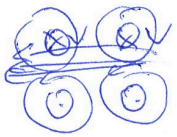
$$= \frac{\mu_0 I_1}{4\pi R^2} |l_1| - \frac{\mu_0 I_2}{4\pi R^2} |l_2|$$

$$= \frac{\mu_0}{4\pi R^2} (I_1 l_1 - I_2 l_2)$$

Př. Ukážeme Ampérova z. $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{celk}$
3.1.6. určete vztah mezi indukcí a mag. polem solenoidu.

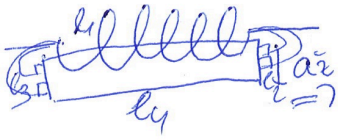


cívka, kterou teče proud



$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{celk} = \int_{c_1} B\,dl + \int_{c_2} \vec{B}\cdot d\vec{l} + \int_{c_3} \vec{B}\cdot d\vec{l}$$

c_2: $B \perp l$; c_3: $B \perp l$



až do ∞ ⇒ spodní str. má 0 příspěvek

$$+ \int_{c_4} \vec{B}\cdot d\vec{l} = \int_{c_4} B\,dl$$

$c_4 \to \infty \to \vec{B} = \vec{0}$

$$\Rightarrow B l_1 = \mu_0 N I$$

$n = \frac{N}{l_1}$ hustota smyček

$$= \mu_0 n l_1 I$$

$$B = \mu_0 n I$$

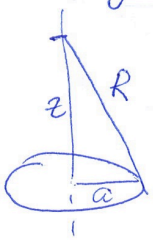
$$\vec{A} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{l}}{R} \qquad \vec{B} = \mathrm{rot}\,\vec{A}$$

$\vec{A}$ ... vektorový potenciál

(obdobně jako $\varphi = \frac{1}{4\pi\varepsilon_0} \int \frac{dQ}{R}$ $\vec{E} = -\nabla\varphi$)

Př. 3.1.13. Spočítejte mag. pole na ose závitu





$R^2 = z^2 + a^2$

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{l}\times\vec{R}}{R^3}$$

my máme jen B ve směru osy z

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{dl \cdot R \cdot \sin\alpha}{R^2}$$

$$= \frac{\mu_0 I}{4\pi} \cdot \frac{1}{R^2} \cdot \sin\alpha \cdot \int dl = \frac{\mu_0 I}{4\pi} \cdot \frac{1}{R^2} \cdot 2\pi a \cdot 1 = \frac{\mu_0 I}{2R^2} \cdot a$$

$\sin\alpha = 1$

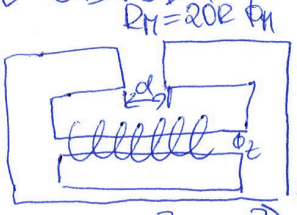
$\alpha = 90°$



$$B_z = B\cos\beta = B \cdot \frac{a}{R} = \frac{\mu_0 I a^2}{2R^3}$$

MAGNETICKÉ OBVODY

Př. 3.2.2.



$\vec{B} = \mu \vec{H}$ (jako $\vec{D} = \varepsilon \vec{E}$)

Najděte intenzitu mag. pole v mezeře.

$\oint \vec{B} \cdot d\vec{l} = \mu I$ | $\mathcal{F} = \oint \vec{H} d\vec{l} = NI$ ← úbytek magnetomotorického napětí ← poč. zdroj jako $\mathcal{E}$

$\oint \vec{H} \cdot d\vec{l} = I$ | $\Phi = \int \vec{B} \cdot d\vec{S}$ ← mag. tok I

$R_m = \int \frac{dl}{\mu S}$ ← mag. odpor R

$U = \int \vec{E} d\vec{l}$ | $U = RI$

$U_m = \int \vec{H} d\vec{l}$ | $U_m = R\Phi$

$\vec{B}$ se při $\vec{S} = \text{konst}$ nemění, mění se $\vec{H}$

$\mathcal{E} = RI$

$\mathcal{F} = R_m \Phi = NI$

platí Kirchhoffovy zákony

~~$\Phi_2 R + \Phi_M 2R$~~ $\Phi_2 = \Phi_M + \Phi_1$

$\Phi_1 R + \Phi_2 R = \mathcal{F}$

$\Phi_2 R + (2\Theta + R)\Phi_M = \mathcal{F}$ ($2\Theta + R = R_M$)

$\Phi_2 R + \Phi_M 2R = \Phi_1 R + \Phi_2 R$

$\Phi_M = \frac{\Phi_1 R}{2R} = \frac{\Phi_1}{2}$

$\Phi_M = \frac{\mathcal{F} - \Phi_2 R}{2R} = \frac{\mathcal{F} - (\Phi_M + \Phi_1)R}{2R}$

$0 = \mathcal{F} - 2R\Phi_M - \Phi_M R - \Phi_1 R$

$\Phi_M = \frac{\mathcal{F} - \Phi_1 R}{2 2R}$

$\Phi_M = \frac{\mathcal{F} - 2\Phi_M R}{22R}$

$22R\Phi_M = \mathcal{F} - 2\Phi_M R$

$43\Phi_M R = \mathcal{F}$

$\Phi_M = \frac{\mathcal{F}}{43R}$

$H_M d = 2\Theta R \cdot \frac{\mathcal{F}}{43R} I$ ($2\Theta R = R$, I)

$\mathcal{E}$

$H_M = \frac{20\mathcal{F}}{43d}$

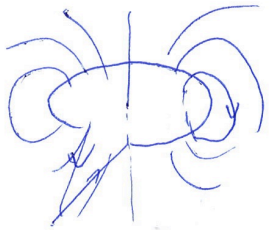
$\int \vec{H} \cdot d\vec{l} = \int_d H dl = Hd$

jinak: $\Phi = \int \vec{B} d\vec{S} = \int B dS = BS = \frac{\mathcal{F}}{43R}$

$R_M = \frac{d}{\mu S}$

$\mu H S = \frac{\mathcal{F}}{43R} \Rightarrow H = \frac{\mathcal{F}\mu S}{\mu S \cdot 43 \cdot d}$

poznámka k minulému cv.



znali jsme směr, proto jsme místo

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{l} \times \vec{R}}{R^3}$$

počítali

$$B = \frac{\mu_0 I}{4\pi} \int \frac{dl \sin\alpha}{R^2}$$

mohli jsme počítat

$$B_z = \frac{\mu_0 I}{4\pi} \int \frac{dl \sin\alpha \; a}{R^2 \sqrt{a^2+z^2}}$$ průmět na osu z

což vyjde stejné jako to, co jsme počítali minule.

Další př. se dal spočítat z $\oint \vec{H} d\vec{l} = \phi_2 \, 2R_m$

$$H = \frac{2\mathcal{F}}{4\pi d} \qquad \vec{B} = \mu \vec{H}$$

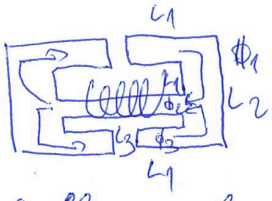
nebo $\oint \vec{B} d\vec{S} = BS$

$$\mu H S = \frac{\mathcal{F}}{4\pi R_m}$$

$$R_m = \frac{R_M}{2} = \frac{d}{\mu_0 S \cdot 2}$$

$$H = \frac{\mathcal{F} \mu_0 S 2}{4\pi d \mu_0 S} = \frac{2\mathcal{F}}{4\pi d}$$

Př. 3.2.5.



Kolik A musí být v závitech, aby v mezerách bylo pole B. Známe L_1, S.

$$\mathcal{F} = \oint \vec{H} \cdot d\vec{l} = NI$$

$$R_m = \int \frac{dl}{\mu S} = \frac{l}{\mu S}$$

$$\Phi_2 = \Phi_1 + \Phi_3 \qquad \Phi_1 = \Phi_2$$

$$NI = \Phi_1 \left(\frac{L_1}{\mu S} + \frac{L_2}{\mu S} + \frac{L_3}{\mu_0 S} \right) + 2\Phi_1 \frac{L_1}{\mu S}$$

$$NI = 3\Phi_1 \frac{L_1}{\mu S} + \Phi_1 \frac{L_2}{\mu S} + \Phi_1 \frac{L_3}{\mu_0 S} \qquad \Phi_1 = BS$$

$$NI = \cancel{S} B \cancel{S} \left(3\frac{L_1}{\mu \cancel{S}} + \frac{L_2}{\mu \cancel{S}} + \frac{L_3}{\mu_0 \cancel{S}} \right) \Rightarrow I = \frac{B}{N} \left(3\frac{L_1}{\mu} + \frac{L_2}{\mu} + \frac{L_3}{\mu_0} \right).$$

Př. 3.3.3

$$\mathcal{E} = -\frac{\partial \Phi}{\partial t}$$

↳ elektromotorické napětí

$\Phi = \int \vec{B} \cdot d\vec{S}$

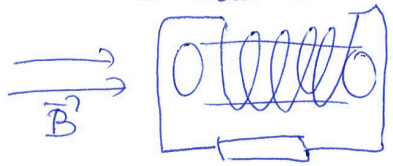
- mění se B
- mění se S

mění se Φ

$\Phi = LI$ jako $Q = CU$

↑ tok ↑ indukčnost

trubka z izolačního materiálu



N závitů
$\vec{B}$ homog. mag. pole
známe D - průměr cívky

máme spočítat proudový impulz, ke kterému dojde při otočení o 180°

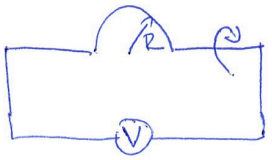
$\Phi_1 = BS = B \cdot \pi \left(\frac{D}{2}\right)^2 N$

$\Phi_2 = -BS = B\pi \left(\frac{D}{2}\right)^2 N$

př. impulz je $\int I dt$ $\quad \mathcal{E} \Delta t = \Delta \Phi$

napěťový impulz je tedy $\Phi_2 - \Phi_1 = \pi B \frac{D^2}{2} N$

Př. 3.3.5. pevný drát se otáčí v homog. mag. poli s frekvencí f.
Voltmetr ukazuje napětí U.
Jaká je mag. indukce B?



$\omega = 2\pi f$

$$\Phi = \int \vec{B}\,d\vec{S} = \int B\cos\varphi\,dS = \int B\cos\omega t\,dS$$

$$= \int B\cos(2\pi f t)\,dS = B\cos(2\pi f t)\,S$$

$$= B\cos(2\pi f t)\,\frac{\pi R^2}{2}$$

$$\mathcal{E} = -\frac{\partial \Phi}{\partial t} = +\frac{B\pi R^2}{2}\cdot\sin(2\pi f t)\cdot 2\pi f = B\pi^2 R^2 f \sin(2\pi f t)$$

Voltmetr změří časovou střední hodnotu (nestíhá se měnit)

$$U_{2ft}^2 = \frac{1}{T}\int_0^T u^2\,dt$$

$$\int_0^T \sin^2 t + \int_0^T \cos^2 t = \int_0^T 1$$

$$A + A = T$$

$$A = \frac{T}{2}$$

$$\int_0^T \sin^2 t\,dt = A = U_{2ft}^2 = \frac{T}{2}$$

$$U_{2ft} = \frac{T}{\sqrt{2}} \qquad U_{2ft} = \frac{U_m}{\sqrt{2}}$$

$$U_{2ft}^2 = \frac{\pi^4 R^4 B^2 f^2}{T^2}\underbrace{\int_0^T \sin^2(2\pi f t)\,dt}_{\frac{T}{2}} = \frac{\pi^4 R^4 B^2 f^2}{2}$$

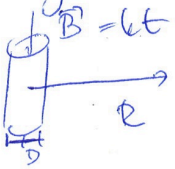
$$U_{2ft} = \frac{\pi^2 R^2 B f}{\sqrt{2}}$$

amplituda proudu (vn. odpor Voltmetru je r)

$$I_{max} = \frac{\pi^2 R^2 B f}{r}$$

Př. 3.3.12 železný ~~kruhový~~ plný válec

$\vec{B} = kt$



elektrického

největší intenzitu el. pole ve vzd. R

$$\int_A^B \vec{E}\cdot d\vec{l} = U_{AB}$$

$$\oint_A \vec{E}\cdot d\vec{l} = 0$$ v elstat. v mag. ne

$$\oint \vec{E}\cdot d\vec{l} = -\frac{\partial \int \vec{B}\,d\vec{S}}{dt}$$

el. pole je buď



nebo

$$\int \mathrm{rot}\,\vec{E}\cdot d\vec{S} = -\frac{\partial \int B\,dS}{dt}$$

$$\mathrm{rot}\,\vec{E} + \frac{\partial \vec{B}}{\partial t} = \vec{0}$$

$$E\,2\pi R = \frac{-\partial\left(kt\pi\frac{D^2}{4}\right)}{\partial t} = -k\pi\frac{D^2}{4}$$

$$E = -\frac{kD^2}{8R}$$

nemůže, rozpor s Gauss. V.

$$\Phi_1 = L_1 \overset{I}{\Phi_1} + M_{12} \overset{I}{\Phi_2}$$
$$\Phi_2 = M_{21} \overset{I}{\Phi_1} + L_2 \overset{I}{\Phi_2}$$

vzájemná indukčnost | vlastní indukčnost

$$Q_1 = C_{11}\varphi_1 + C_{12}\varphi_2$$
$$Q_2 = C_{21}\varphi_1 + C_{22}\varphi_2$$

C_{21} influenční koef. | C_{11}, C_{22} kapacitní koef.

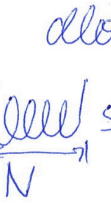
$\varphi = A\,Q$ (A potenciálové koef.)

pro 1 smyčku: $\Phi = LI$

$$\mathcal{E} = -\frac{\partial \Phi}{\partial t} = -L\frac{\partial I}{\partial t}$$

$\oint \vec{E}\cdot d\vec{l} = 0$ platí i pro stacionární el. pole

Př. 3.4.7. dlouhá válcová cívka. Chceme vyjádřit pro indukčnost.



$$\Phi = LI \Rightarrow L = \frac{\Phi}{I} = \frac{\mu I N^2 S}{l \cdot I} = \boxed{\frac{\mu N^2 S}{l}}$$

$$\frac{\Phi}{N} = \oint \vec{B}\cdot d\vec{S} = BS = \frac{\mu I N}{l}\cdot S$$

pro solenoid platí: $\vec{B} = \frac{\mu I N}{l}$

Př. 3.4.1. $\vec{A}(\vec{R}) = \frac{\mu}{4\pi}\int \frac{\vec{i}(\vec{r}')}{R}\,dV$ — pro vodič, kterým protéká proud $\vec{A}(R) = \frac{\mu I}{4\pi}\int \frac{d\vec{l}}{R}$

$\vec{R} = \vec{r} - \vec{r}'$

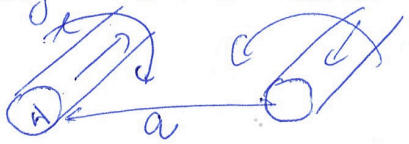
$$L = \frac{\Phi}{I} = \frac{\int \vec{B}\cdot d\vec{S}}{I} = \frac{\int \mathrm{rot}\,\vec{A}\cdot d\vec{S}}{I} = \frac{\oint \vec{A}\cdot d\vec{l}}{I}$$

$\vec{B} = \mathrm{rot}\,\vec{A}$

$$= \frac{\mu_0 I}{4\pi I}\oint \frac{d\vec{l}\cdot d\vec{l}}{R}$$

Př. 3.4.3. dvě smyčky, dokažte, že $M_{12} = M_{21}$ — jedno $d\vec{l}$ vede u jedné cívky a druhé $d\vec{l}$ u druhé

Př. 3.4.11. vzájemná indukčnost dvou vodičů



1. ~~vlastní indukčnost~~ indukčnost

$$W = \frac{1}{2}LI^2 = \frac{1}{2}\int \vec{B}\cdot\vec{H}\,dV$$

B nekonečně dlouhého vodiče: $\oint \vec{B}\cdot d\vec{l} = \mu_0 I$

$B\cdot 2\pi R = \mu_0 I$

$B = \frac{\mu_0 I}{2\pi R}$

Kde plochou mezi vodiči na jednotku délky

$$M = \frac{\Phi}{I} =$$

$$\Phi_L = \frac{\Phi}{l} = \int_r^{r+a} \frac{\vec{B}\,d\vec{S}}{l} = \frac{1}{l}\int_{r_0}^{r+a}\int \frac{\mu_0 I}{2\pi R}\cdot d\vec{l}\cdot d\vec{R}$$

$$= \int_r^{r+a} \frac{\mu_0 I}{2\pi R}\cdot d\vec{R} = \frac{\mu_0 I}{2\pi}\int_r^{r+a}\frac{1}{R}\,d\vec{R}$$

$$= \frac{\mu_0 I}{2\pi}\left[\log|R|\right]_r^{r+a}$$

$$= \frac{\mu_0 I}{2\pi}\cdot\log\frac{r+a}{r}$$ — příspěvek 1 vodiče

jsou 2 a příspěvky se sčítají $\Rightarrow \Phi = 2\frac{\mu_0 I}{2\pi}\cdot\log\frac{r+a}{r}$

$$= \frac{\mu_0 I}{\pi}\log\frac{r+a}{r}$$

$$M = \frac{\mu_0 I}{\pi I}\log\frac{r+a}{r} = \frac{\mu_0}{\pi}\log\frac{r+a}{r}$$

$W = \int_0^t UI\,dt$

pro kondenzátor: $W = \int_0^Q u\,dQ = \frac{1}{C}\int Q\,dQ = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QU = \frac{1}{2}CU^2$

$\frac{dQ}{dt} = I \qquad U\,dt = -L\,dI$

$W_m = \int UI\,dt = L\int_0^I I\,dI = \frac{1}{2}LI^2 = \frac{1}{2}I\Phi = \frac{1}{2}\frac{\Phi^2}{L}$

analogicky $W = \frac{1}{2}\int \vec{E}\cdot\vec{D}\,dV \qquad W_m = \frac{1}{2}\int \vec{B}\cdot\vec{H}\,dV$

v předchozím případě $B = \mu H$

$W_m = \frac{1}{2}\int_0^V \frac{B^2}{\mu}V = \frac{1}{2}\int_0^R \frac{\mu^2 I^2}{4\pi^2}\cdot\frac{R^2}{r^4}\cdot\frac{1}{\mu}\,l\,2\pi R\,dR = \frac{1}{2}\cdot\frac{\mu I^2}{4\pi}\cdot\frac{l2\pi}{r^4}\int_0^r R^3\,dR$

$= \frac{\mu I^2 l}{16\pi}$

$dV = l\,2\pi R\,dR$

$W_m = \frac{\mu I^2 l}{16\pi} = \frac{1}{2}LI^2$

$L_{vlk.} = \frac{2\mu I^2 l}{16\pi} = \frac{\mu l}{8\pi}$

$L_1 = \frac{L_{vlk.}}{l} = \frac{\mu}{8\pi}$

$L = 2\cdot L_1 = \frac{\mu}{4\pi}$

$L = \frac{\mu}{4\pi}\left(1 + 4\log\left(\frac{r+a}{r}\right)\right)$

$\oint \vec{B}\cdot d\vec{l} = \mu I \cdot \frac{\pi R^2}{\pi r^2}$

$B\,2\pi R = \mu I\,\frac{R^2}{r^2}$

$B = \frac{\mu I}{2\pi}\,\frac{R}{r^2}$

Př. 4.1.11. Indukčnost na jednotku délky koaxiálního vedení



$L = \frac{\mu_0}{4\pi}\left(\frac{1}{2} + 2\log\left(\frac{r_2}{r_1}\right)\right)$

proud teče jen jedním $\longrightarrow \cdot\frac{1}{2}$

od r do r+a $\longrightarrow$ od r_1 do r_2

Př. 3.5.2 energie mag. pole v toroidu



$\vec{B} =$

$\oint \vec{B}\cdot d\vec{l} = \mu_0 I$

$B\,2\pi R = \mu_0 I N$

$B = \frac{\mu_0 I N}{2\pi R}$

toroid



$B = \frac{\mu_0 NI}{2\pi r} = \frac{\mu_0 NI}{\ell}$ ~ mimo

Př. toroidní cívka ze dvou vinutí

$W = \frac{1}{2}\Phi I = \frac{1}{2}LI^2 = \frac{1}{2}\frac{\Phi^2}{L}$

2N závitů $\rightarrow \frac{1}{2}N\Phi I = \frac{1}{2}LI^2 = \frac{1}{2}\frac{(N\Phi)^2}{L}$

$\Phi = \int_S \vec{B}\cdot\vec{n}\,dS = B\cdot S$

každý závit produkuje $\vec{B} = \frac{\mu_0 2NI}{\ell}$

$W_{2N} = \frac{1}{2}\underbrace{\frac{\mu_0 2NI}{\ell}}_{B}\cdot S\,(2NI) = 2\frac{\mu_0 N^2 I^2}{\ell}\cdot S$

jak se změní W_1, jestliže jedno vinutí odpojíme?

$W_N = \frac{1}{2}\frac{\mu_0 NI}{\ell}\cdot S\cdot NI = \frac{1}{2}\frac{\mu_0 N^2 I^2}{\ell}\cdot S$

interakční energie obou cívek:

$W_{2N} - 2W_N = \frac{2}{2}\cdot\frac{\mu_0 N^2 I^2}{\ell}\cdot S$

Př. 3.5.8. Nosnost elektromagnetu



$F = -\frac{dW}{dx}$ $\quad \vec{F} = -\mathrm{grad}\,W$ $\quad W = \frac{1}{2}\int \vec{B}\vec{H}\,dV$

$\mathcal{F} = \Phi R_m$ $\quad \mathcal{F} = \oint \vec{H}\,d\vec{\ell} = NI$

$NI = BS\left(\frac{\ell}{\mu S} + \frac{2\ell_M}{\mu_0 S}\right)$ $\quad R_m = \frac{\ell}{\mu S}$

$B = \frac{NI}{S\left(\frac{\ell}{\mu S} + \frac{2\ell_M}{\mu_0 S}\right)} = \frac{NI}{\frac{\ell}{\mu} + \frac{2\ell_M}{\mu_0}}$

$W = \frac{1}{2}\int \vec{B}\cdot\vec{H}\,dV = \frac{1}{2}\int \frac{B^2}{\mu}\,dV$ $\quad H = \frac{B}{\mu}$

$= \frac{1}{2}\frac{B^2}{\mu}S(\ell + 2\ell_M)$

$F = -\frac{dW}{d\ell_M} = -\frac{d}{d\ell_M}\left(\frac{B^2}{2\mu}S(\ell + 2\ell_M)\right) = -\frac{B^2 S}{\mu} = \frac{\Phi^2}{\mu S}$

Př. 3.5.11. $\vec{E}\parallel\vec{B}$, $\vec{E} = \mathrm{konst}$, $\vec{B} = \mathrm{konst}$ $\quad \vec{F}_L = q\left[\vec{E} + (\vec{v}\times\vec{B})\right]$

nabitá částice hm. m, náb. q. Popište dráhu částice.

$\vec{E} = (E_x, 0, 0)$ $\quad \vec{B} = (B_x, 0, 0)$

$v_z = \sin\omega t$

$v_y = \cos\omega t$

$m\frac{dv_x}{dt} = q\left[E_x + 0\right] = qE$

$m\frac{dv_y}{dt} = q\left[0 + v_z B_x\right] = +v_z qB$

$m\frac{dv_z}{dt} = q\left[0 - v_y B_x\right] = -v_y qB$

$m \cdot (-\sin \omega t) \cdot \omega = Bqv_z = Bq \cdot \sin \omega t \Leftrightarrow -m\omega = Bq$

$m \cdot (\cos \omega t) \cdot \omega = -Bqv_y = -Bq \cos \omega t \Leftrightarrow m\omega = -Bq$

$\omega = \frac{-Bq}{m}$

$\int \frac{m\, dv_x}{dt} = \int q E \, dt$

$m v_x = qEt$

$v_x = \frac{qEt}{m} = \frac{Ft}{m} = at$

STŘÍDAVÉ PROUDY

$U - L\frac{dI}{dt} = IR + \frac{Q}{C}$ $\quad \int I\,dt$

$C = \frac{Q}{U}$ $\quad \frac{dQ}{dt} = I$

$dQ = I\,dt$

$I = I_0 \cos(\omega t) = \mathrm{Re}[I_0 e^{i\omega t}]$

$U = RI = RI_0 e^{i\omega t}$

$U = \frac{1}{C}\int I\,dt = \frac{I_0 e^{i\omega t}}{C\, i\omega} = \frac{-iI_0 e^{i\omega t}}{C\omega} = \left(\frac{i}{C\omega}\right) I_0 e^{i\omega t} = \frac{e^{-i\frac{\pi}{2}}}{C\omega} I_0 e^{i\omega t} = \frac{I_0}{C\omega} e^{i(\omega t - \frac{\pi}{2})}$ (kapacitance)

$U = L\frac{dI}{dt} = (i L\omega) I_0 e^{i\omega t} = e^{i(\omega t + \frac{\pi}{2})} I_0 L\omega$ (indukctance)

$e^{i\frac{\pi}{2}} = i$

$e^{i\cdot\frac{\pi}{2}} = -i$



$Z_R = R$

$Z_L = i\omega L$

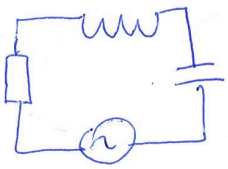
$Z_C = -\frac{i}{\omega C}$

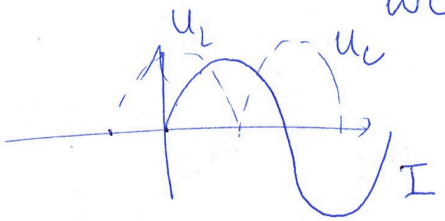
odpor $= R$

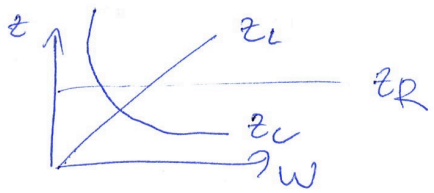
indukčance $= \omega L e^{i(\omega t + \frac{\pi}{2})}$

kapacitance $= e^{-i\frac{\pi}{2}(\omega t - \frac{\pi}{2})} \frac{}{\omega C}$

Př.







$z = z_R + z_L + z_C = R + i\left(\omega L - \frac{1}{\omega C}\right)$

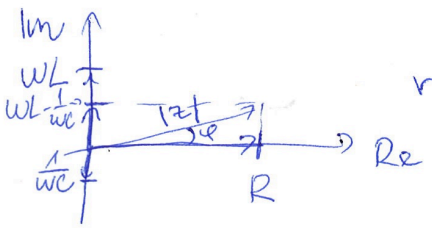
$|z| = \sqrt{Re^2 + Im^2}$

$\varphi = \mathrm{arctg}\, \frac{Im\, z}{Re\, z}$

$I = I_0 e^{i\omega t}$

$U = z_L I = \omega L e^{i\frac{\pi}{2}} I_0 e^{i\omega t}$

reálná část $\rightarrow$ $Re(U) = \omega L I_0 \cos(\omega t + \frac{\pi}{2})$

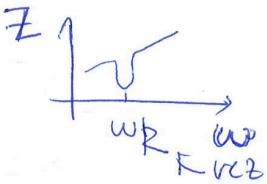


na cívce a kapacitě se energie nestrácí, nebo se přeměňuje na jiné formy. Ztrácí se na odporu

$P = UI$ $\quad Re(P) = UI\cos\varphi = \frac{1}{2}U_0 I_0 \cos\varphi = U_{eff} I_{eff} \cos\varphi$

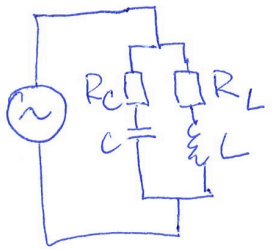
$U_{eff}^2 = \frac{1}{T}\int_0^T u^2 dt$ $\quad = RI^2 = \frac{U^2}{R}$

Rezonance: $Im(Z) = 0$ (navzájem se vyruší induktance a kapacitance)



Obvod přenáší nejvíc proudu

Př. 5.2.19



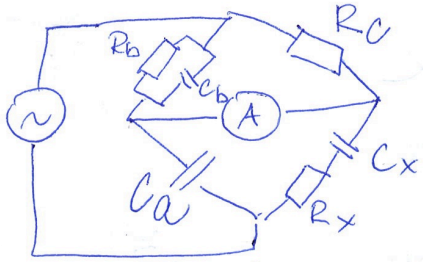
$$\frac{1}{Z} = \frac{1}{R_C + C} + \frac{1}{R_L + L}$$

$$Z = \frac{(R_C + C)(R_L + L)}{R_L + R_C + L + C} = \frac{(R_C + \frac{i}{\omega C})(R_L + i\omega L)}{R_L + R_C - \frac{i}{\omega C} + i\omega L}$$

rozdělíme na Im a Re část, dostaneme zlomek tvaru $\frac{a+bi}{c+di} \to \frac{c-di}{c-di}$ a

pak položíme $Im\, z = 0$.

Př. 5.2.19. Nalezněte podmínky rovnováhy



Určete R_x a C_x, aby $A = 0$.

$$Z_b = Z_c$$
$$Z_a = Z_x$$
$$\Rightarrow \frac{Z_b}{Z_a} = \frac{Z_c}{Z_x}$$

$$Z_a = \frac{-i}{\omega C_a}$$

$$\frac{1}{Z_b} = \frac{1}{R_b} + \frac{1}{Z_B} = \frac{1}{R_b} - \frac{\omega C_B}{i} = \frac{i - \omega C_B R_B}{R_b i} = \frac{1 + i\omega C_B R_B}{R_B}$$

$$Z_B = \frac{-i}{\omega C_B}$$

$$Z_c = R_C$$
$$Z_x = R_x + \cancel{\omega L} - \frac{i}{\omega C_x} = \frac{R_x \omega C_x - i}{\omega C_x}$$

$$\Rightarrow \frac{\frac{R_B}{1 + i\omega C_B R_B}}{\frac{-i}{\omega C_a}} = \frac{R_C}{\frac{R_x \omega C_x - i}{\omega C_x}}$$

$$\frac{R_B}{1 + i\omega C_B R_B} \cdot \frac{R_x \omega C_x - i}{\cancel{\omega C_x}} = -R_C \cdot \frac{i}{\cancel{\omega C_a}}$$

$$C_a R_B R_x \omega C_x - i R_B C_a = -i R_C C_x + R_C C_x \omega C_B R_B$$

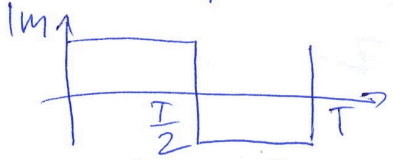
$C_a R_B R_x \omega C_x = R_C C_x \omega C_B R_B$

$R_B C_a = R_C C_x \Rightarrow C_x = \frac{R_B C_a}{R_C}$

$R_x = \frac{R_C C_B}{C_a}$

Př. 5.3.2.

Stanovte střední a efektivní hodnotu proudu:

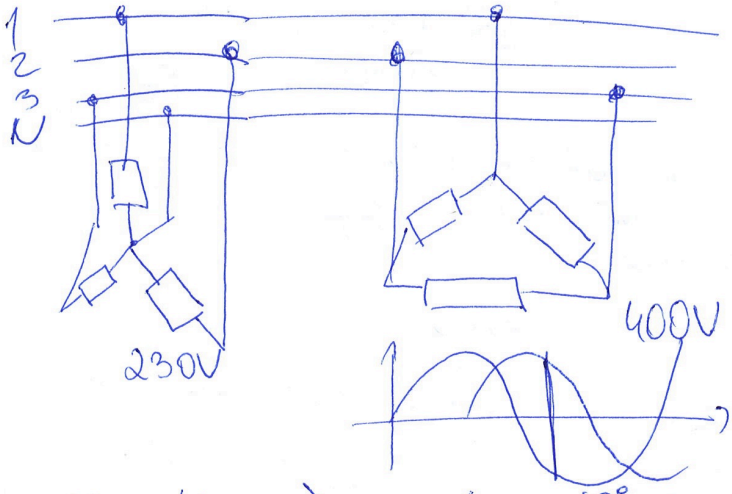


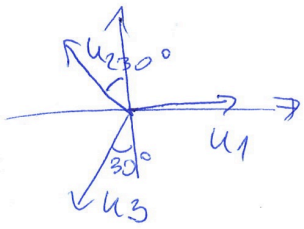
jen dosazení xd

$I_{eff}^2 = \frac{1}{T}\int_0^T I^2 dt$

$I_{st.} = \frac{1}{T}\int_0^T I\,dt$

Př.





$u_1 = (1 - i0)u_m = u_m e^{-i0°}$

$u_2 = (-\frac{1}{2} + i\frac{\sqrt{3}}{2})u_m = u_m e^{-i120°}$

$u_{21} = u_2 - u_1 = (-\frac{3}{2} + i\frac{\sqrt{3}}{2})u_m$

$|u_{21}| = u\sqrt{(-\frac{3}{2})^2 + (\frac{\sqrt{3}}{2})^2} = u\sqrt{\frac{12}{4}} = \sqrt{3}$

$\sqrt{3}\,u = 400V$